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Heavy-to-light form factors in the quark model with heavy infrapropagators.

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Abstract. We calculate the heavy-to-light form factors in the relativistic quark model with heavy infrapropagators. Their q^2-dependence in the physical region is defined by two parameters: the "infrared" parameter ν characterizing the infrared behavior of the heavy quark and the mass difference of the heavy meson and heavy quark $E = m_H - M_Q$.

It is found that the values of the $D \to K(K^*)$ and $D \to \pi(\rho)$ form factors at $q^2 = 0$ are in excellent agreement with the available experimental data and other approaches whereas these values for $B \to \pi(\rho)$ transitions are found to be larger than those of several other models.

The obtained form factors are used to calculate the widths of the semileptonic decays of B and D mesons. The comparison of our results with the available experimental data and other approaches is given.

1 Introduction

A theoretical study of the heavy-to-light transitions is more complicated than heavy-to-heavy ones because of the heavy quark symmetry cannot be applied in this case. The results of nonperturbative calculations made within lattice QCD [1]-[6], QCD sum rules [7]-[10], and various quark models [11]-[13] are seemed to be rather different each other to give a reliable conclusion about the behavior of the form factors in the physical region. Nevertheless, such calculations may be useful for our understanding what is really important under description of the heavy-to-light transitions.

One has to remark that the most of mentioned approaches allows one to calculate the values of the form factors at some fixed point of q^2, the mo-

mentum transfer squared, and then to adopt some simple shape for getting q^2 dependence in the full physical region.

The lattice calculations give the form factors near $q^2 = q^2_{\max}$ and then extrapolate them to $q^2 = 0$ using polelike dependence. The QCD sum rules method allows one to give the reliable results at $q^2 = 0$ and then also the pole dominance hypothesis is used for extrapolating them to physical region. The q^2-dependence have been obtained by using the QCD sum rules method in the paper [10]. It was found that the pole dominance dependence is a good approximation for the vector form factors whereas this approximation is not applicable for the axial form factors. In the nonrelativistic constituent quark model [11] the form factors, computed with the overlap integral of the nonrelativistic meson wave functions, have the exponential q^2 dependence. The quasipotential approach [12] predicts similar behavior. The q^2 dependence of the relevant form factors was derived from the independent quark model [13] with a scalar-vector-harmonic potential. Some nonrelativistic [14] and relativistic [15, 16, 17] quark models adopt polelike (monopole, dipole,...) ansatz for the q^2 dependence of the form factors.

The dispersion relations incorporating with the Heavy Quark Effective Theory (HQET) and Chiral Perturbation Theory (ChPT) [18], with the light-cone model [19], with the crossing symmetry and QCD perturbative calculation in unphysical kinematic region [20] have been used for studying the form factors. In the paper [21] heavy-light form factors have been analyzed within approach incorporating HQET, ChPT and Vector Meson Dominance (VMD) model.

Therefore it may be seen that a completely consistent calculation of the heavy-to-light form factors in the full physical region of the momentum transfer squared is a problem not only for the approaches based on the first principles of QCD but also for various quark models. The polelike q^2 dependence usually used for the form factors should be at least justified from the more deep physical representations.

The present work is devoted to the calculation of the form factors describing the heavy-to-light transitions within a relativistic quark model with confined light quarks and infrared heavy quark propagators [22].

Our previous approach [23, 24] was based upon the quark confinement model (QCM) which incorporates the confinement of light quarks by devising a quark propagator that has no singularities, thus forbidding the production of a free quark [24]. This model then allows one to perform covariant calculations of Feynman diagrams with dressed light quark propagators free of simple pole. It was shown that the QCM approach successful in many static and non-static properties of the light hadrons [24]. In the paper [22] the infrared behavior of the heavy quark has been taken into account by modifying its conventional propagator in terms of a single parameter ν. The weak decay constants and the Isgur-Wise function have been calculated.

Here we slightly modify the inclusion of the infrared behavior of the heavy quark in a such way to provide the conservation of local properties of Feynman diagrams like gauge invariance and the Ward-Takachashi identity. We investigate the dependence of the heavy-to-light form factors on two parameters: the

infrared parameter ν and the mass difference of the heavy meson and heavy quark $E = m_H - M_Q$.

We found that the values of the $D \to K(K^*)$ and $D \to \pi(\rho)$ form factors at $q^2 = 0$ are in excellent agreement with the available experimental data and other approaches whereas these values for $B \to \pi(\rho)$ transitions are found to be larger those of several other models.

The obtained form factors are used to calculate the widths of the semileptonic decays of B and D mesons. We compare our results with the available experimental data and other approaches.

2 Model

Our approach is based on the effective interaction Lagrangian which describes the transition of hadron into quarks. For example, the transition of heavy meson H into heavy Q and light q quarks may be described by

$$\mathcal{L}_{\rm int}(x)=g_H H(x)\int dx_1\int dx_2 \delta\left[x-\frac{M_Q x_1+m_q x_2}{M_Q+m_q}\right] f\left[(x_1-x_2)^2\right]\bar{Q}(x_1)\varGamma_H\lambda_H q(x_2)+\text{h.c.} \tag{1}$$

Here, λ_H and $\varGamma_H$ are the Gell-Mann and Dirac matrices, respectively, which provide the flavor and spin numbers of mesons H. The function $f(x^2)$ characterizes the smearing of interaction between a hadron and quarks.

The coupling constants g_H defined by what is usually called the *compositeness condition* which means that the renormalization constant of the meson field is equal to zero:

$$Z_H = 1 - 3g_H^2/(2\pi)^2 \tilde{\varPi}'_H(m_H^2) = 0 \tag{2}$$

where $\tilde{\varPi}'_H$ is the derivative of the meson mass operator.

It is readily seen that in the heavy quark limit $M_Q \gg m_q$ the Eq.(1) becomes as

$$\mathcal{L}_{\rm int}(x) \Rightarrow g_H H(x)\bar{Q}(x)\varGamma_H\lambda_H \int dx_2 f\left[(x-x_2)^2\right] q(x_2) + \text{h.c.} \tag{3}$$

which means the light degrees of freedom are factorized out from the heavy ones in according to the heavy quark symmetry. One has to remark that the function $f(x^2)$ characterizing the long distance contributions is related to the light quark only in this limit. It leades to the modification of the light quark propagator in the momentum space:

$$\frac{1}{m_q - \not{p}} \Longrightarrow \frac{f^2(p^2)}{m_q - \not{p}} \tag{4}$$

In the paper [25] the monopole function $f(p^2) = 1/(\varLambda^2 - p^2)$ has been used for calculations of physical values. But in this case the physical amplitudes had the threshold corresponding to quark production. In our approach [24] an entire (nonpole) function has been used for a single light quark propagator to ensure

the quark confinement.

$$\frac{1}{m_q - \not p} \Rightarrow \int \frac{d\sigma_z}{\Lambda z - \not p} = G(\not p) = \frac{1}{\Lambda}\left[a(-\frac{p^2}{\Lambda^2}) + \frac{\not p}{\Lambda} b(-\frac{p^2}{\Lambda^2})\right] \tag{5}$$

with the functions a and b being defined by

$$a(-p^2) = \int \frac{z d\sigma_z}{z^2 - p^2} \qquad b(-p^2) = \int \frac{d\sigma_z}{z^2 - p^2}. \tag{6}$$

To conserve the local properties of Feynman diagrams like the Ward identities, we had used the following prescription for the modification of a line with n-light quarks within the Feyman diagram:

$$\prod_{i=0}^{n} \frac{1}{m_q - \not p_i} \Gamma_i \Rightarrow \int d\sigma_z \prod_{i=0}^{n} \frac{1}{\Lambda z - \not p_i} \Gamma_i \tag{7}$$

In QCM we had used a simple choice of the confinement functions [24]

$$a(u) = a_0 \exp(-u^2 - a_1 u) \qquad b(u) = b_0 \exp(-u^2 + b_1 u). \tag{8}$$

The parameters a_i, b_i, and Λ have been determined from the best model description of hadronic properties at low energies and the following values were found [24]:

$$a_0 = b_0 = 2 \qquad a_1 = 1 \qquad b_1 = 0.4 \qquad \text{and} \qquad \Lambda = 460 \text{ MeV},$$

which describe various physical observable quite well [24].

Since a heavy quark (with mass M_Q) in a heavy meson is under the influence of soft gluons (which sets the scale $\Lambda_{QCD} << M_Q$), it may be regarded as nearly on its mass-shell where the infrared regime should take place for its propagation. The infrared behavior for one-fermion Green's function (propagator) has been investigated in various papers (see, for instance, [26] and the references therein). The result is well-known only for abelian gauge theories:

$$S(p) \sim (m - \not p)^{-1-\nu}, \tag{9}$$

where $\nu = (\alpha_S/4\pi)(3 - \lambda)$ with λ being the gauge parameter.

We have chosen [22] the heavy quark propagator as in Eq.(9) to calculate the weak decay constants and the Isgur-Wise function.

But as it is readily seen the using of such kind of propagators in the Feynman diagrams destroys some local relations like Ward identity which based on the obvious equality

$$\frac{1}{m - \not k - \not p} \not p \frac{1}{m - \not k} = \frac{1}{m - \not k - \not p} - \frac{1}{m - \not k}$$

In this paper we suggest to take into account the infrared behavior of the heavy quarks by slightly different way which nevertheless allows one to conserve

the local properties of the Feynman diagrams. We shall use the infrapropagator for a single heavy quark line in a diagram:

$$S^h(\not k + \not p) = \frac{\Lambda^\nu}{(M_Q - \not k - \not p)^{1+\nu}} = -\Lambda^\nu \frac{\sin(\pi\nu)}{\pi\nu} \int\limits_0^\infty \frac{du}{u^\nu} \frac{\partial}{\partial u} \frac{1}{M_Q + u - \not k - \not p}$$
$$= \int d\sigma_u^h S_u^h(\not p + \not k) \tag{10}$$

Here M_Q is the constituent mass of a heavy quark and

$$d\sigma_u^h = -\Lambda^\nu \frac{\sin(\pi\nu)}{\pi\nu} \frac{du}{u^\nu} \frac{\partial}{\partial u} \quad ; \quad S_u^h(\not k + \not p) = \frac{1}{M_Q + u - \not k - \not p} \tag{11}$$

But we will use the prescription for a line with n heavy quarks analogous to those used in Eq.(7) for averaging of light quarks:

$$\prod_{i=0}^{n} \frac{1}{M_{Q_i} - \not p_i} \Gamma_i \Rightarrow \int d\sigma_u^h \prod_{i=0}^{n} \frac{1}{M_{Q_i} + u - \not p_i} \Gamma_i \tag{12}$$

Let $K^{(0)}(M_{Q_1}, ..., M_{Q_n})$ be the structural integral corresponding to the Feynman diagram with n heavy quark local propagators. It is readily seen that the taking into account the infrared behavior according to Eqs.(10,11,12) gives

$$K^{(\nu)}(M_{Q_1},..., M_{Q_n}) = \int d\sigma_u^h K^{(0)}(M_{Q_1}+u,..., M_{Q_n}+u)$$
$$= -\frac{\sin(\pi\nu)}{\pi\nu} \int\limits_0^\infty \frac{du}{u^\nu} \frac{\partial}{\partial u} K^{(0)}(M_{Q_1}+u,..., M_{Q_n}+u) = \frac{\sin(\pi\nu)}{\pi\nu} \Big\{ K^{(0)}(M_{Q_1},..., M_{Q_n})$$
$$+\nu \int\limits_0^1 \frac{du}{u^\nu} \frac{K^{(0)}(M_{Q_1},..., M_{Q_n}) - K^{(0)}(M_{Q_1}+u,..., M_{Q_n}+u)}{u}$$
$$-\nu \int\limits_1^\infty \frac{du}{u^{\nu+1}} K^{(0)}(M_{Q_1}+u,..., M_{Q_n}+u) \Big\} \approx \frac{\sin(\pi\nu)}{\pi\nu} \Big\{ K^{(0)}(M_{Q_1}, ..., M_{Q_n})$$
$$+\nu \int\limits_0^1 \frac{du}{u^\nu} \frac{K^{(0)}(M_{Q_1}, ..., M_{Q_n}) - K^{(0)}(M_{Q_1}+u, ..., M_{Q_n}+u)}{u} \Big\} \tag{13}$$

This means that we can calculate the integral corresponding to Feynman diagram with the local heavy quark propagators and then apply the averaging according to Eq.(13) to take into account the infrared behavior.

3 Heavy-to-light form factors

First, we introduce the necessary notation. We will write P for light pseudoscalar meson and V for light vector meson with the masses m_P and m_V respectively. We will write H for heavy pseudoscalar meson with a mass m_H.

In what follows we assume that all masses and momenta in the structural integrals are given in units of $\Lambda = 460$ MeV.

The invariant amplitudes for $H \to l\nu$ and $H \to P(V)l\nu$ transitions are defined as

$$A(H \to e\nu) = \frac{G}{\sqrt{2}} V_{qq'} (\bar{e} O_\mu \nu) M_H^\mu(p) \tag{14}$$

$$A(H \to Pe\nu) = \frac{G}{\sqrt{2}} V_{qq'} (\bar{e} O_\mu \nu) M_{HP}^\mu(p, p') \tag{15}$$

$$A(H \to Ve\nu) = \frac{G}{\sqrt{2}} V_{qq'} (\bar{e} O_\mu \nu) \epsilon_\nu M_{HV}^{\mu\nu}(p, p'), \tag{16}$$

where

$$M_H^\mu(p) = \frac{3}{4\pi^2} g_H \int d\sigma_u^h \int d\sigma_z^l \int \frac{d^4k}{4\pi^2 i} \mathrm{tr}\Big\{ i\gamma^5 S_u^h(\not k + \not p) O^\mu S_z^l(\not k) \Big\} = i f_H p^\mu \tag{17}$$

$$\begin{aligned} M_{HP}^\mu(p, p') &= \frac{3}{4\pi^2} g_H g_P \int d\sigma_u^h \int d\sigma_z^l \int \frac{d^4k}{4\pi^2 i} \mathrm{tr}\Big\{ i\gamma^5 S_u^h(\not k + \not p) O^\mu S_z^l(\not k + \not p') i\gamma^5 S_z^l(\not k) \Big\} \\ &= f^+(q^2)(p^\mu + p'^\mu) + f^-(q^2)(p^\mu - p'^\mu) \end{aligned} \tag{18}$$

$$\begin{aligned} M_{HV}^{\mu\nu}(p, p') &= \frac{3}{4\pi^2} g_H g_V \int d\sigma_u^h \int d\sigma_z^l \int \frac{d^4k}{4\pi^2 i} \mathrm{tr}\Big\{ i\gamma^5 S_u^h(\not k + \not p) O^\mu S_z^l(\not k + \not p') \gamma^\nu S_z^l(\not k) \Big\} \\ &= -i(m_H + m_V) A_1(q^2) g^{\mu\nu} + i \frac{A_2(q^2)}{m_H + m_V} (p^\mu + p'^\mu) p^\nu + i \frac{A_0(q^2)}{m_H + m_V} (p^\mu - p'^\mu) p^\nu \\ &+ \frac{2V(q^2)}{m_H + m_V} e^\mu_{\nu\alpha\beta} p^\alpha p'^\beta \end{aligned} \tag{19}$$

Here

$$q^2 = (p - p')^2, \qquad p^2 = m_H^2, \qquad p'^2 = m_{P(V)}^2 \tag{20}$$

The integrals like (17), (18), (19) are calculated by using the standard Feyman α – parametrization and the prescriptions (6) and (13) to integrate over $d\sigma_z^l$ and $d\sigma_u^h$, respectively. The explicit expressions for the form factors are given in Appendix.

The hadron-quark coupling constants for light pseudoscalar and vector mesons and heavy pseudoscalar mesons determined from the compositeness condition (2) are written down

$$g_P = \frac{2\pi}{\sqrt{3}} \sqrt{\frac{2}{R_P(m_P^2)}}, \qquad g_V = 2\pi \sqrt{\frac{1}{R_V(m_V^2)}}, \qquad g_H = \frac{2\pi}{\sqrt{3}} \sqrt{\frac{1}{R_H(m_H^2)}} \tag{21}$$

The functions $R_i(x)$ are shown in Appendix.

4 Numerical results and discussions

We have two adjustable parameters in our approach: the difference between the heavy meson and heavy quark masses $E = m_D - M_c = m_B - M_b$ and the parameter ν characterizing the infrared behavior of heavy quark. We will adjust them by fitting the available experimental data for branching ratios of D-meson decays. The results of fit are given in Table 1. One has to remark that the allowed regions for the adjustable parameters are quite narrow: the parameter E varies from 0.315 up to 0.415 GeV whereas the infrared parameter ν varies from 0.6 up 0.7. In what follows we will give our results for E=0.365 GeV and ν=0.65. This set of the parameters seems to be best for describing the available experimental data. Our results for f_D and f_B in this case are

$$f_D = 193 \text{ MeV}, \qquad f_B = 136 \text{ MeV} \tag{22}$$

The magnitudes of form factors of the $D \to K(K^*)e\nu$ at $q^2 = 0$ are given in Table 2. One can see that our results are in good agreement with the experimental data and other approaches. The dependence on the momentum transfer squared is shown in Fig.1 and Fig.2.

The results for the form factors and decay widths of B-meson are given in Tables 3 and 4 by using the values of adjustable parameters obtained above. It is seen that the values of the form factors at $q^2 = 0$ are substantially larger those obtained in QCD Sum Rules [8]– [10] and quark models [12]-[15] but close to the results of papers [6], [7], [20], [21] and [27] for $f^+(0)$. In the latter it was found that using the heavy quark symmetry and assuming pole dominance for the form factors, $f_+^{B\pi}(0)$ is estimated to be ≈ 0.39. If the requirement of heavy quark symmetry is weaken so that it applies only to soft pion emissions from the heavy meson, one finds $f_+^{B\pi}(0) \approx 0.53$.

Recently, the CLEO collaboration [29] claims the following results for semileptonic B decays branching ratios and $|V_{ub}|$:

$$\begin{aligned} &\mathrm{Br}(B^0 \to \pi^- l^+ \nu) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4} \\ &\mathrm{Br}(B^0 \to \rho^- l^+ \nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4} \\ &|V_{ub}| = (3.3 \pm 0.2^{+0.3}_{-0.4} \pm 0.7) \times 10^{-3}, \end{aligned} \tag{23}$$

One has to note that the branching fractions for $B \to \pi e\nu$ and $B \to \rho e\nu$ have been obtained by using a variety of models. What is given in Eq. (23) is just the average value of the branching fractions obtained for each model. The errors are statistical, systematic and the estimated model dependence. The values for $|V_{\mathrm{ub}}|$ were extracted from the branching fractions using $\tau_{B_0} = 1.56 \pm 0.05$ ps. To obtain $|V_{\mathrm{ub}}|_{\mathrm{aver}}$, the π and ρ modes were combined by fixing their ratio to the prediction for each model.

This means that it is not easy to use these results for checking predictions of other models. Nevertheless, we extract the values $\Gamma(B \to \pi(\rho)e\nu)/|V_{ub}|^2 \times 10^{12} s^{-1}$ from Eq. (23) and put them without errors in Table 4. As one expects our predictions are almost twice larger those values obtained in a such manner.

At the same time the ratio π/ρ is in a good agreement with the experimental result.

The q^2 dependence of form factors is shown in Fig. 3.

One can see from Fig.1-3 that the behavior of the form factors $f^+(q^2)$ and $V(q^2)$ in the physical regions of the momentum transfer squared ($q^2 < 1.5$ Gev^2 for the D-meson decay and $q^2 < 20.0$ Gev^2 for the B-meson decay) agrees quite well with q^2 dependence of the monopole form factor. At the same time the behavior of the form factors $A_1(q^2)$ and $A_2(q^2)$ is rather different from the monopole function and similar to the prediction of QCD sum rules [10].

5 Conclusion

We have slightly modified our previous work [22] to take into consideration the infrared behavior of the heavy quark in consistent way to provide the conservation of local properties of Feynman diagrams like gauge invariance and the Ward-Takachashi identity. The infrared behavior of the heavy quark is modelled in terms of a single parameter ν which modifies the simple free Feynman propagator.

We have investigated the dependence of the heavy-to-light form factors calculated within this approach on two parameters: the infrared parameter ν and the mass difference of the heavy meson and heavy quark $E = m_H - M_Q$.

We found that the values of the $D \to K(K^*)$ or $D \to \pi(\rho)$ form factors at $q^2 = 0$ are in excellent agreement with the available experimental data and other approaches whereas these values for $B \to \pi(\rho)$ transitions are found to be larger than those of several other models.

We have used the obtained form factors to calculate the widths of the semileptonic decays of B and D mesons.

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Appendix

Two-point function:

$$
\begin{aligned}
f_P &= g_P \, \frac{3\Lambda}{(2\pi)^2}\left\{A_0 + \frac{m_P^2}{4}\int\limits_0^1 d\alpha \, \mathbf{a}\left(-\alpha\frac{m_P^2}{4}\right)\sqrt{1-\alpha}\right\}, \\
R_P(x) &= B_0 + \frac{x}{4}\int\limits_0^1 du b(-\frac{ux}{4})\frac{(1-u/2)}{\sqrt{1-u}},
\end{aligned}
$$

$$R_V(x) = B_0 + \frac{x}{4}\int_0^1 du b(-\frac{ux}{4})\frac{(1-u/2+u^2/4)}{\sqrt{1-u}},$$

$$R_H(x) = \int d\sigma_u^h \int_0^1 d\alpha\alpha\left[M\mathbf{a}\left(\delta(\alpha,x)\right) + \left(\frac{3\alpha-2\alpha^2}{2(1-\alpha)^2}M^2 - \frac{1}{2}\alpha x\right)\mathbf{b}\left(\delta(\alpha,x)\right)\right]$$

Heavy-to-light form factors:

$$f_H = g_H\frac{3\Lambda}{(2\pi)^2}\int d\sigma_u^h\int_0^1 d\alpha\alpha\left(\frac{M^2}{(1-\alpha)^2} - m_H^2\right)\left[(1-\frac{\alpha}{2})\mathbf{a}\left(\delta(\alpha,m_H^2)\right) + M\frac{\alpha}{2}\mathbf{b}\left(\delta(\alpha,m_H^2)\right)\right]$$

$$\begin{aligned} f^+(q^2) &= g_H g_P\frac{3}{2(2\pi)^2}\int d\sigma_u^h\left\{B_0 + \frac{m_P^2}{4}\int_0^1 d\alpha\mathbf{b}\left(-\alpha\frac{m_P^2}{4}\right)\sqrt{1-\alpha} + \int_0^1 d\alpha_1\int_0^1 d\alpha_2\left[M\mathbf{a}\left(\Delta(\alpha_1,\alpha_2)\right)\right.\right. \\ &+ \left.\left.\left((1-\alpha_1)(1-\alpha_2)m_H^2 - M^2 + \alpha_2(1-\alpha_1)q^2\right)\mathbf{b}\left(\Delta(\alpha_1,\alpha_2)\right)\right]\right\} \end{aligned}$$

$$\begin{aligned} f^-(q^2) &= -f^+(q^2) + g_H g_P\frac{3}{(2\pi)^2}\int d\sigma_u^h\left\{\int_0^1 d\alpha_1\int_0^1 d\alpha_2\,\alpha_2(1-\alpha_1)m_P^2\mathbf{b}\left(\Delta(\alpha_1,\alpha_2)\right)\right. \\ &+ \left.\int_0^1 d\alpha\,\alpha(1-\frac{\alpha}{2})\left(\frac{M^2}{(1-\alpha)^2} - q^2\right)\mathbf{b}\left(\delta(\alpha,q^2)\right)\right\} \end{aligned}$$

$$\begin{aligned} A_1(q^2) &= g_H g_V\frac{3}{(2\pi)^2(m_H+m_V)}\int d\sigma_u^h\left\{\int_0^1 d\alpha\alpha\left(\frac{M^2}{(1-\alpha)^2} - q^2\right)\mathbf{b}\left(\delta(\alpha,q^2)\right)\right. \\ &+ \int_0^1 d\alpha_1\int_0^1 d\alpha_2\frac{m_H^2+m_V^2-q^2}{2}\mathbf{a}\left(\Delta(\alpha_1,\alpha_2)\right) \\ &+ \int_0^1 d\alpha_1\int_0^1 d\alpha_2\left[\alpha_1\left(1-\frac{1}{2}\alpha_1\right)\left(\frac{M^2}{(1-\alpha_1)^2} - m_H^2\right) - \alpha_1\left(1-2\alpha_2+\alpha_1\alpha_2\right)\frac{m_H^2+m_V^2-q^2}{2}\right. \\ &- \left.\left.\alpha_2\left(1-\alpha_1+\alpha_1\alpha_2-\frac{1}{2}\alpha_1^2\alpha_2\right)\right]\left[\mathbf{a}\left(\Delta(\alpha_1,\alpha_2)\right) - M\mathbf{b}\left(\Delta(\alpha_1,\alpha_2)\right)\right]\right\} \end{aligned}$$

$$\begin{aligned} A_2(q^2) &= g_H g_V\frac{3(m_H+m_V)}{2(2\pi)^2}\int d\sigma_u^h\int_0^1 d\alpha_1\int_0^1 d\alpha_2\left[\left(1-3\alpha_1+2\alpha_1\alpha_2+2\alpha_1^2-2\alpha_1^2\alpha_2\right)\mathbf{a}\left(\Delta(\alpha_1,\alpha_2)\right)\right. \\ &+ \left.\alpha_1\left(1-2\alpha_1-2\alpha_2+2\alpha_1\alpha_2\right)M\mathbf{b}\left(\Delta(\alpha_1,\alpha_2)\right)\right] \end{aligned}$$

$$A_0(q^2) = -A_2(q^2) - g_H g_V \frac{3(m_H+m_V)}{(2\pi)^2} \int d\sigma_u^h \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \left[2\alpha_1(1-\alpha_1)\,\mathbf{a}\Big(\Delta(\alpha_1,\alpha_2)\Big) \right.$$

$$\left. + \; 2\alpha_1^2 M\mathbf{b}\Big(\Delta(\alpha_1,\alpha_2)\Big)\right]$$

$$V(q^2) = g_H g_V \frac{3(m_H+m_V)}{2(2\pi)^2} \int d\sigma_u^h \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \left[\alpha_1 M\mathbf{b}\Big(\Delta(\alpha_1,\alpha_2)\Big) + (1-\alpha_1)\mathbf{a}\Big(\Delta(\alpha_1,\alpha_2)\Big)\right]$$

Here

$$M = M_Q + u$$

$$A_n = \int_0^\infty du a(u) u^n, \qquad B_n = \int_0^\infty du b(u) u^n, \qquad \delta(\alpha, x) = \frac{\alpha}{(1-\alpha)} M^2 - \alpha x$$

$$\Delta(\alpha_1,\alpha_2) = \frac{\alpha_1}{(1-\alpha_1)} M^2 - \alpha_1(1-\alpha_2) m_H^2 - (1-\alpha_1)(1-\alpha_2)\alpha_2 m_{P(V)}^2 - \alpha_1\alpha_2 q^2$$

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Table 1. Branching ratios for semileptonic and nonleptonic B and D meson decays for various values of the model parameters E and ν.

Decay	Experimental average [28]	$E = 0.415$ $\nu = 0.60$	$E = 0.365$ $\nu = 0.65$	$E = 0.315$ $\nu = 0.70$
$Br(D^0 \to K^- e^+ \nu_e)$	3.8 ± 0.22	4.26	3.77	3.32
$Br(D^0 \to \pi^- e^+ \nu_e)$	$0.39^{+0.23}_{-0.12}$	0.48	0.42	0.35
$Br(D^0 \to K^{*-} e^+ \nu_e)$	2.0 ± 0.4	1.39	1.31	1.23
$Br(D^+ \to \bar{K}^0 e^+ \nu_e)$	6.6 ± 0.9	11.0	9.82	8.57
$Br(D^+ \to \pi^0 l^+ \nu_l)$	0.57 ± 0.22	0.63	0.55	0.47
$Br(D^+ \to \bar{K}^{*0} e^+ \nu_e)$	4.8 ± 0.5	3.54	3.34	3.14
$Br(D^+ \to \rho e^+ \nu_e)$	< 0.37	0.35	0.32	0.29

Table 2. The predictions for the form factors at $q^2 = 0$ in the decay $D \to K(K^*)e\nu$ along with those of other approaches and the experiment.

$D \to K(K^*)e\nu_e$	$f^+(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
Expt.average	0.75 ± 0.04	0.56 ± 0.04	0.4 ± 0.08	1.1 ± 0.2
Our results	0.78	0.52	0.4	1.07
BKS[1]	$0.90^{\pm 8}_{\pm 21}$	$0.83^{\pm 14}_{\pm 28}$	$0.59^{\pm 14}_{\pm 24}$	$1.43^{\pm 45}_{\pm 49}$
LMMS[2]	0.63 ± 0.08	0.53 ± 0.03	0.19 ± 0.21	0.86 ± 0.1
ELC[3]	$0.60^{\pm 15}_{\pm 7}$	0.64 ± 0.16	$0.4^{\pm 28}_{\pm 4}$	0.86 ± 0.24
APE[4]	0.78 ± 0.08	0.67 ± 0.11	0.49 ± 0.34	1.08 ± 0.22
UKQCD[5]	0.67^{+7}_{-8}	0.70^{+7}_{-10}	0.66^{+10}_{-15}	1.01^{+30}_{-13}
GSS[6]	$0.71^{+12}_{-12}{}^{+10}_{-7}$	$0.61^{+6}_{-6}{}^{+9}_{-7}$	$0.83^{+20}_{-20}{}^{+12}_{-8}$	$1.34^{+24}_{-24}{}^{+19}_{-14}$
DP[7]	0.75 ± 0.05			
BBD[10]	0.60^{+15}_{-10}	0.5 ± 0.15	0.6 ± 0.15	1.1 ± 0.25
ISGW[11]	$0.76 - 0.82$	0.8	0.8	1.1
FGM[12]	0.73 ± 0.07	0.63 ± 0.06	0.43 ± 0.04	0.62 ± 0.06
BD[13]	0.8	0.77	1.48	1.32
AW[14]	0.7	0.8	0.6	1.5
BSW[15]	0.76	0.88	1.15	1.3
J[17]	0.78	0.66	0.43	1.04
C[21]	0.67	0.48	0.27	0.95

Table 3. The predictions for the form factors at $q^2 = 0$ in the decay $B \to \pi(\rho)e\nu$ along with those of other approaches.

$B{\to}\pi(\rho)e\nu_e$	$f^+(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
Our results	0.53	0.50	0.51	0.70
ELC[3]	$0.30^{\pm14}_{\pm5}$	0.22±0.05	$0.49^{\pm21}_{\pm5}$	0.37±0.11
APE[4]	0.35±0.08	0.24±0.12	0.27±0.80	0.53±0.31
UKQCD[5]	0.23±0.02	$0.27^{+7}_{-4}{}^{+3}_{-3}$	$0.28^{+9}_{-6}{}^{+4}_{-5}$	
GSS[6]	$0.50^{+14}_{-14}{}^{+7}_{-5}$	$0.16^{+4}_{-4}{}^{+22}_{-16}$	$0.72^{+35}_{-35}{}^{+10}_{-7}$	$0.61^{+23}_{-23}{}^{+9}_{-6}$
DP[7]	0.4±0.1			
N[8]	0.23±0.02	0.38±0.04	0.45±0.05	0.45±0.05
CS[9]	0.24			
B[10]	0.26±0.02	0.5±0.1	0.4±0.2	0.6±0.2
ISGW[11]	0.09	0.05	0.02	0.27
FGM[12]	0.21±0.02	0.26±0.03	0.30±0.03	0.29±0.03
BSW[15]	0.33	0.28	0.28	0.33
J[17]	0.27	0.26	0.24	0.35
M[19]	0.29∼0.2	0.26∼0.17	0.24∼0.16	0.34∼0.22
B[20]	0.38±0.3			
C[21]	0.89	0.21	0.2	1.04
Ch.[27]	0.53			

Table 4. Semileptonic decay rates $\Gamma(B \to \pi(\rho)e\nu)$ in units $|V_{ub}|^2 \times 10^{12}s^{-1}$. The comparison of our results with those of other approaches.

Decay	$B{\to}\pi e\nu_e$	$B{\to}\rho e\nu_e$	$\Gamma(B{\to}\rho e\nu_e)/\Gamma(B{\to}\pi e\nu_e)$
CLEO[29]	(10.59)	(14.72)	$1.4^{+0.6}_{-0.4}\pm0.3\pm0.4$
Our results	19.0	25.0	1.32
ELC[3]	9±6	14±12	(1.55)
APE[4]	8±4		
DP[7]	14.5±0.59		
N[8]	3.6±0.6	5.1±1.0	(1.41)
B[10]	5.1±1.1	12±4	(2.3)
ISGW[11]	2.1	8.3	3.9
FGM[12]	3.1±0.6	5.7±1.2	(1.83)
BSW[15]	7.4	26	3.5
J[17]	10.0	19.1	1.91
M[19]	7±2	10.6±3.2	1.45±0.1
C[21]	54.0	34.0	0.63

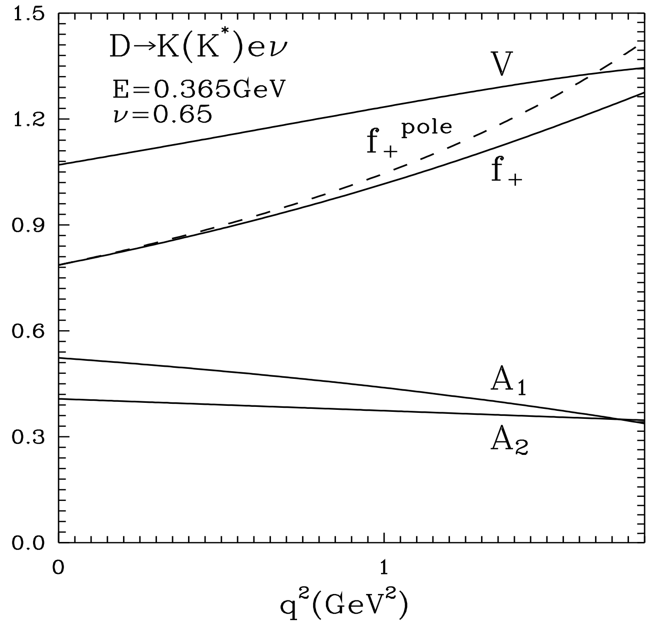

Figure 1. q^2-dependence of the form factors relevant for the decays $D \to K(K^*)e\nu$.

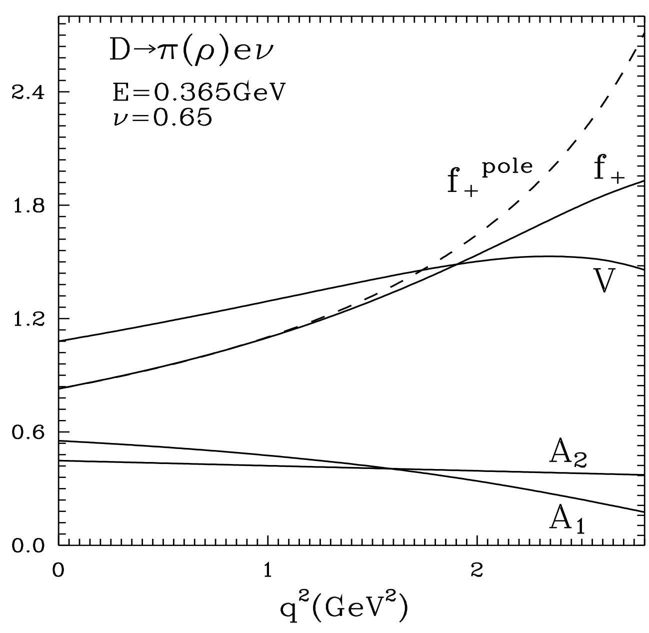

Figure 2. q^2-dependence of the form factors relevant for the decays $D \to \pi(\rho)e\nu$.

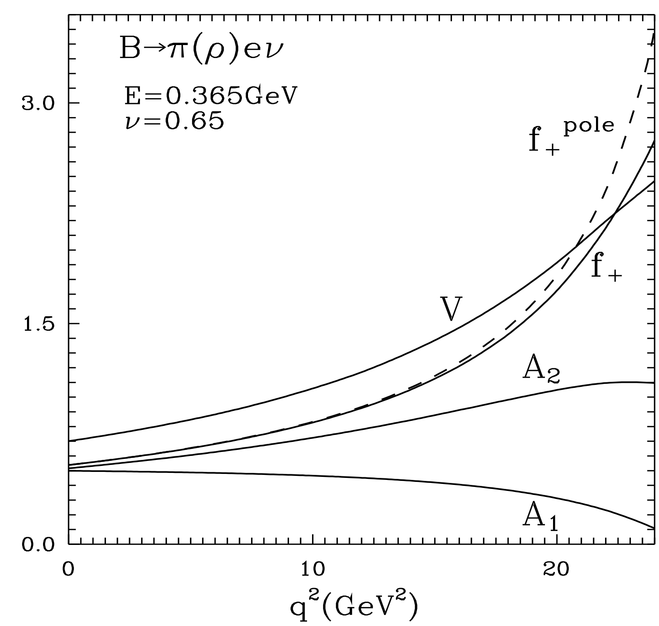

Figure 3. q^2-dependence of the form factors relevant for the decays $B \to \pi(\rho)e\nu$.